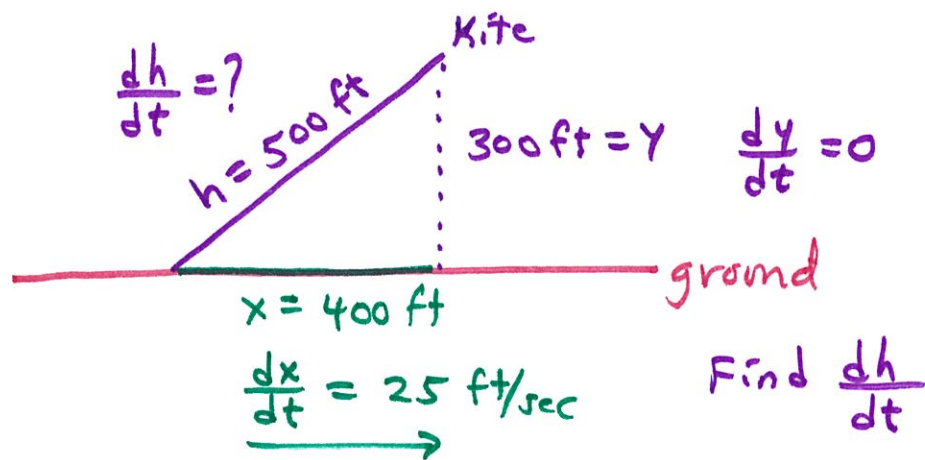


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14.



$$x^2 + y^2 = h^2$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2h \cdot \frac{dh}{dt}$$

$$400(25) + 300(0) = 500 \frac{dh}{dt}$$

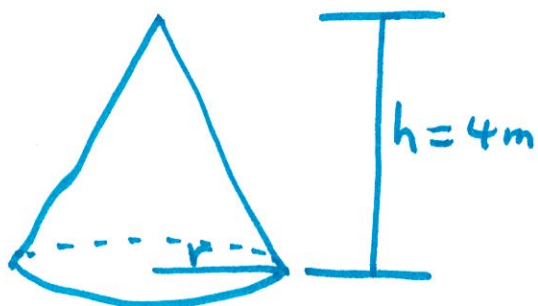
$$400(25) = 500 \frac{dh}{dt}$$

$$\frac{400(25)}{500} = \frac{dh}{dt}$$

$$20 = \frac{dh}{dt}$$

Inge must let out the string at a rate of 20 ft/sec

16.



$$\frac{dV}{dt} = 10 \text{ m}^3/\text{min}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V(r) = \frac{1}{3} \pi r^2 \left(\frac{3}{4} r \right)$$

$$h = \frac{3}{8} d \quad h = 4 \text{ m}$$

$$h = \frac{3}{8} (2r)$$

$$h = \frac{3}{4} r \quad 4 = \frac{3}{4} r$$

$$\frac{16}{3} = r$$

a. $V(r) = \frac{1}{4} \pi r^3$

$$\frac{dV}{dt} = \frac{3}{4} \pi r^2 \frac{dr}{dt}$$

$$10 = \frac{3}{4} \pi \left(\frac{16}{3} \right)^2 \frac{dr}{dt}$$

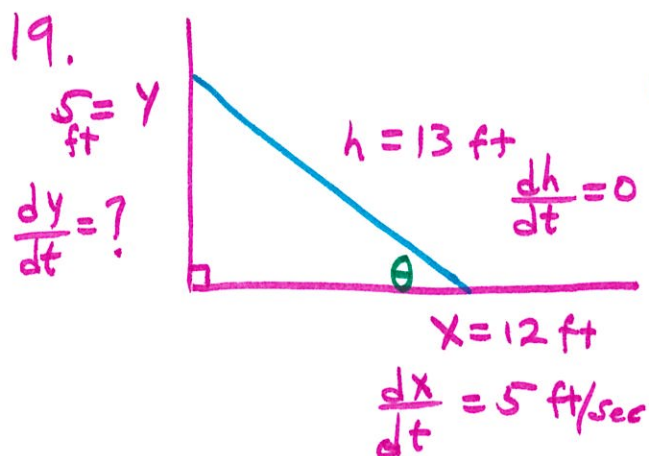
b. $10 = \frac{64}{3} \pi \frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{30}{64\pi} \text{ m/min}$$

$$\frac{dr}{dt} = \frac{15}{32\pi} \text{ m/min} \approx 0.149 \text{ m/min}$$

$$h = \frac{3}{4} r \quad \frac{dh}{dt} = \frac{3}{4} \frac{dr}{dt} \quad \frac{dh}{dt} \approx \frac{3}{4} (14.9) \approx 11.18 \text{ cm/min}$$

$$\approx 14.9 \text{ cm/min}$$



(a)

$$x^2 + y^2 = h^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2h \frac{dh}{dt}$$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = h \frac{dh}{dt}$$

$$12(5) + 5 \frac{dy}{dt} = 13(0)$$

$$\frac{dy}{dt} = \frac{-12(5)}{5}$$

$$\frac{dy}{dt} = -12 \text{ ft/sec}$$

The ladder is sliding down the wall at a rate of 12 ft/sec

(b)

$$A = \frac{1}{2}xy \quad \frac{dA}{dt} = \frac{1}{2} \frac{dx}{dt}y + \frac{1}{2}x \frac{dy}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2}(5)(5) + \frac{1}{2}(12)(-12)$$

$$= 12\frac{1}{2} - 72$$

$$= 59\frac{1}{2} \text{ ft}^2/\text{sec}$$

The area is changing (increasing) at a rate of $59\frac{1}{2} \text{ ft}^2/\text{sec}$.

(c)

$$\tan \theta = \frac{y}{x} \quad \text{Find } \frac{d\theta}{dt}$$

$$\sec^2 \theta \cdot \frac{d\theta}{dt} = \frac{\frac{dy}{dt} \cdot x - y \cdot \frac{dx}{dt}}{x^2}$$

$$\left(\frac{13}{12}\right)^2 \frac{d\theta}{dt} = \frac{(-12)(12) - (5)(5)}{144}$$

$$\frac{d\theta}{dt} = \frac{-169}{144} \cdot \frac{144}{169}$$

$$\frac{d\theta}{dt} = -1 \text{ radian/sec.}$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right)$$

$$\theta \approx 0.3948 \text{ radians}$$

27. Surface area of a sphere is $A = 4\pi r^2$

$$d = 20 \text{ cm} \quad r = 10 \text{ cm} \quad \frac{dV}{dt} = -8 \text{ mL/min} = -8 \text{ cm}^3/\text{min}$$

$$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$
$$-8 = 4\pi (10)^2 \frac{dr}{dt}$$

$$\frac{-8}{400\pi} = \frac{dr}{dt}$$

$$\frac{-1}{50\pi} = \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{-1}{50\pi} \text{ cm/min}$$

$$A = 4\pi r^2 \quad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 8\pi (10) \left(\frac{-1}{50\pi} \right)$$

$$\frac{dA}{dt} = -\frac{8}{5} \text{ cm}^2/\text{min}$$

$$\frac{dA}{dt} = -1.6 \text{ cm}^2/\text{min}$$

The surface Area is decreasing at the rate $1.6 \text{ cm}^2/\text{min}$